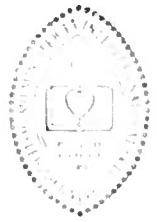


Office of Naval Research Contract Nonr-610 (06)
Task Order NR 064-476



LEHIGH UNIVERSITY INSTITUTE FOR RESEARCH

AD 634 356

Technical Report No. 1

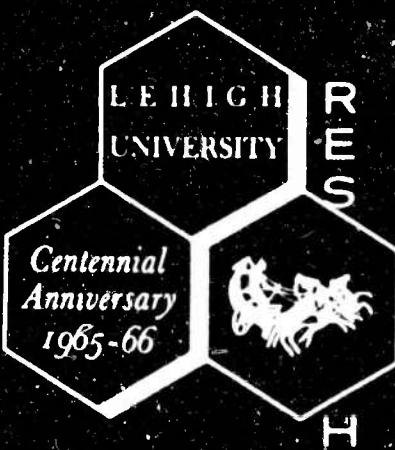
ON THE WESTERGAARD METHOD OF CRACK ANALYSIS

CLEARINGHOUSE FOR FEDERAL SCIENTIFIC AND TECHNICAL INFORMATION			
Hardcopy	Microfiche		
\$1.00	\$.50	11 pp as	
ARCHIVE COPY 1			

by
G. C. Sih

D D C
March 1966

Department of Applied Mechanics
Lehigh University, Bethlehem, Pennsylvania



**Office of Naval Research
Contract Nonr-610 (06)
Task Order NR 064-476**

Technical Report No. 1

On the Westergaard Method of Crack Analysis

by

G. C. Sih

Department of Applied Mechanics

Lehigh University

Pethlehem, Pennsylvania

March 1966

**Reproduction in whole or in part is permitted by the United
States Government. Distribution of this document is unlimited.**

On the Westergaard Method of Crack Analysis¹

by

G. C. Sih²

A survey of the literature on the analysis of crack problems shows that the Westergaard method [1]³ has been most frequently quoted and used by the practitioners in fracture mechanics for nearly thirty years. Surprisingly enough it has yet to be pointed out that the method in [1] suffers severe restrictions for a class of problems dealing with the infinite medium with a crack (or cracks) subjected to external loads at infinity. These restrictions will be derived in the work to follow from the more general consideration of complex potentials originated by Muskelishvili [2].

In the theory of two-dimensional isotropic elasticity, the stresses and displacements may be expressed in terms of two complex functions $\phi(z)$ and $\psi(z)$ of the variable $z=x+iy$. They are

1 The results presented in this paper were obtained in the course of an investigation carried out under Contract Nonr-610(06) with the Office of Naval Research in Washington, D.C.

2 Professor of Mechanics, Lehigh University, Bethlehem, Pa.

3 Numbers in brackets designate References at end of Note.

$$\sigma_x + \sigma_y = 4 \operatorname{Re} [\phi'(z)] \quad (1)$$

$$\sigma_y - \sigma_x + 2i \tau_{xy} = 2[\bar{z} \phi''(z) + \psi'(z)]$$

and

$$2\mu(u+iv) = \kappa \phi(z) - z \overline{\phi'(z)} - \overline{\psi(z)} \quad (2)$$

where κ takes the value $3-4v$ for plane strain and $(3-v)/(1+v)$ for generalized plane stress and v is the Poisson's ratio. The shear modulus is denoted by μ . Eqs. (1) and (2) may be simplified by introducing symmetry conditions as follows:

Symmetric Problems. If the external loads are placed symmetrically with respect to the x -axis along which the cracks are situated, then the shearing stress τ_{xy} must vanish at $y=0$, i.e.,

$$\operatorname{Im} [\bar{z} \phi''(z) + \psi'(z)] = 0 , \quad \text{for } y = 0 \quad (3)$$

Eq. (3) can be satisfied by taking

$$\psi'(z) + z \phi''(z) + A = 0 \quad (4)$$

where A is a real constant depending upon the applied load. Making use of eq. (4) and letting $\kappa = 3 - 4v$ for plane strain, the stresses take the form

$$\sigma_x = 2 \operatorname{Re} [\phi'(z)] - 2y \operatorname{Im} [\phi''(z)] + A$$

$$\sigma_y = 2 \operatorname{Re} [\phi'(z)] + 2y \operatorname{Im} [\phi''(z)] - A \quad (7)$$

$$\tau_{xy} = -2y \operatorname{Re} [\phi''(z)]$$

and the displacements are

$$2\mu u = 2(1-2v) \operatorname{Re} [\phi(z)] - 2y \operatorname{Im} [\phi'(z)] + Ax$$

(8)

$$2\mu v = 4(1-v) \operatorname{Im} [\phi(z)] - 2y \operatorname{Re} [\phi'(z)] - Ay$$

Hence, the problem is reduced to the determination of a single complex function $\phi(z)$ satisfying the necessary boundary conditions. Eqs. (7) and (8) agree with eqs. (4-6) and (9-10) in [1], respectively, only if

$$2 \phi'(z) = Z, \quad A = 0$$

In general, the constant A cannot be neglected arbitrarily. To illustrate this point, consider the problem of an infinite medium with a central crack of length $2a$ along the x -axis. The boundary conditions are

$$\sigma_y = \tau_{xy} = 0, \quad y = 0, \quad -a < x < a$$

(9)

$$\sigma_x = \epsilon \sigma^\infty, \quad \sigma_y = \sigma^\infty, \quad \tau_{xy} = 0, \text{ as } (x^2 + y^2)^{1/2} \rightarrow \infty$$

The solution to this problem is given by [2]

$$\phi'(z) = (\sigma^\infty/2)[z/(z^2-a^2)^{1/2}] - (1-\epsilon)(\sigma^\infty/4) \quad (10)$$

$$\psi'(z) = (a^2\sigma^\infty/2)[z/(z^2-a^2)^{3/2}] + (1-\epsilon)(\sigma^\infty/2)$$

Inserting eq. (10) into (4), A is found to be

$$A = - (1-\epsilon)(\sigma^\infty/2)$$

Note that A vanishes only in the special case of $\epsilon=1$ corresponding to the case of uniform tension at infinity. The same applies to the problem of an infinite row of collinear cracks spaced periodically in an infinite medium [3].

Skew-Symmetric Problems. For loads applied skew-symmetrically with respect to the crack line, say along the x-axis, the normal stress σ_y is required to vanish at $y=0$, or

$$\text{Re } [2\phi'(z) + \bar{z} \phi''(z) + \psi'(z)] = 0, \text{ at } y = 0 \quad (11)$$

lly.

It follows that

$$\psi'(z) + 2\phi'(z) + z\phi''(z) + iB = 0 \quad (12)$$

where B is a real constant. Substituting eq. (12) into eqs. (1) and (2) and separating the real and imaginary parts give

$$\sigma_x = 4 \text{ Re } [\phi'(z)] - 2y \text{ Im } [\phi''(z)] \quad (13)$$

$$\sigma_y = 2y \text{ Im } [\phi''(z)]$$

$$\tau_{xy} = -2 \operatorname{Im} [\phi'(z) - 2y \operatorname{Re} [\phi''(z)] - B] \quad (13)$$

and

$$2\mu u = 4(1-v) \operatorname{Re} [\phi(z)] - 2y \operatorname{Im} [\phi'(z)] - By \quad (14)$$

$$2\mu v = 2(1-2v) \operatorname{Im} [\phi(z)] - 2y \operatorname{Re} [\phi'(z)] - Bx$$

The Westergaard version of eqs. (13) and (14) may be obtained by selecting an Airy stress function of the form $y \operatorname{Im} Z$. The results are the same as those given above if

$$2 \phi'(z) = Z, \quad B = 0$$

The restriction of $B=0$ leads to a trivial solution for the problem of uniform in-plane shear applied to an infinite medium containing a crack. For this problem, the conditions are

$$\sigma_y = \tau_{xy} = 0, \quad y = 0, \quad -a < x < a \quad (15)$$

$$\sigma_x = \sigma_y = 0. \quad \tau_{xy} = \tau^\infty, \quad \text{as } (x^2 + y^2)^{1/2} \rightarrow \infty$$

From [2], the complex functions are

$$\phi'(z) = -(i\tau^\infty/2)[z/(z^2 - a^2)^{1/2}] + i\tau^\infty/2 \quad (16)$$

$$\psi'(z) = i\tau^\infty[z/(z^2 - a^2)^{1/2}] - i(a^2\tau^\infty/2)[z/(z^2 - a^2)^{3/2}]$$

The constant B may thus be found from eqs. (12) as

$$B = -\tau^{\infty}$$

Hence, B cannot vanish for a non-trivial solution.

It should be mentioned that the Westergaard method is valid for loads applied to the crack surfaces since in such cases the constants A and B have no contribution.

References

1. H.M. Westergaard, "Bearing Pressure and Cracks", Journal of Applied Mechanics, Vol. 6, 1937, pp. A 49-53.
2. N.I. Muskhelishvili, "Some Basic Problems of Mathematical Theory of Elasticity", P. Noordhoff Ltd., Groningen, Holland, 1953.
3. Private communication with I. N. Sneddon.

PART I - GOVERNMENT

Administrative & Liaison Activities

Chief of Naval Research
Attn: Code 102 (Dr. F. J. Weyl)
423
439
468
(2)

Department of the Navy
Washington, D. C. 20360

Commanding Officer
ONR Branch Office
495 Summer Street
Boston, Massachusetts 02210

Commanding Officer
ONR Branch Office
219 S. Dearborn Street
Chicago, Illinois 60604

Commanding Officer
ONR Branch Office
Box 39, Navy 100
c/o Fleet Post Office
New York, New York 09510
(5)

Commanding Officer
ONR Branch Office
207 West 21st Street
New York, New York 10011

Commanding Officer
ONR Branch Office
1030 E. Orson Street
Pasadena, California 91101

Commanding Officer
ONR Branch Office
U.S. Post Office & Courts Bldg.
1076 Mission Street
San Francisco, California 94103

U. S. Naval Research Laboratory
Attn: Technical Information Div.
Washington, D. C. 20390
(6)

Defense Documentation Center
Cameron Station
Alexandria, Virginia 22314
(20)

Army

Commanding Officer
U. S. Army Research Off.-Durham
Attn: Mr. J. J. Murray
CRD-AA-IP
Box CM, Duke Station
Durham, North Carolina 27706

Commanding Officer
AMER-AIL
U. S. Army Materials Res. Agcy.
Watertown, Massachusetts 02172

Redstone Scientific Info. Center
Chief, Document Section
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809

AMSMI-RKP
Attn: Mr. T. H. Duerr
Redstone Arsenal, Alabama 35809

Ballistic Research Laboratories
Attn: Dr. A. S. Elder
Aberdeen Proving Ground
Aberdeen, Maryland 21005

Ballistic Research Laboratories
Attn: Mr. H. P. Gay
AMBR-ID
Aberdeen Proving Ground
Aberdeen, Maryland 21005

Technical Library
Aberdeen Proving Ground
Aberdeen, Maryland 21005

Navy

Commanding Officer and Director
Attn: Code 052 (Cent. Lib. Br.)
050
700 (Struc. Mech. Lab.)
720
725
730 (Mr. W. J. Sette)
901 (Dr. M. Strassberg)
911 (Dr. R. Liebowitz)
915 (Mr. A. O. Sykes)
960 (Mr. E. P. Noonan)
962 (Dr. E. Buchmann)
David Taylor Model Basin
Washington, D. C. 20007

Navy (cont'd.)

Director
Aeronautical Structures Lab.
Naval Air Engineering Center
Naval Base
Philadelphia, Pennsylvania 19112

Director
Attn: Code 5360
5500
6200
6210
6250
6260
Technical Library
Naval Research Laboratory
Washington, D. C. 20360

Chief, Bureau of Naval Weapons
Attn: Code DLL-3
R-12
RAAD-2
RAAD-24 (Mr. E. N. Ryan)
RM
RMSP-2
RMSP-11 (Mr. I. Silver)
RMSP-22 (Mr. J. C. Ardinger)
RR
RRRE
RRRE-61 (Mr. M. J. Marciniak)
RU

Department of the Navy
Washington, D. C. 20360

Chief, Bureau of Ships
Attn: Code 210-L
305
345
421
423
430
440
442
443
1500

Department of the Navy
Washington, D. C. 20360

Commander
U. S. Naval Weapons Laboratory
Dahlgren, Virginia 22448

Bureau of Yards & Docks Tech. Lib.
Yards & Docks Annex
Department of the Navy
Washington, D. C. 20390

Air Force

Commander, WARD
Attn: Code WMWWD
AFPL (PRDS)
Structures Division
APL (MCRA)
Code WMWWD
AFPL (PDT)
Code WMRC
AFPL (MAAM)
Code WMCLY
SAC (AFPSB, Mr. Lakia)
Wright-Patterson Air Force Base
Dayton, Ohio 45433

Commander
Chief, Applied Mechanics Group
U. S. Air Force Inst. of Tech.
Wright-Patterson Air Force Base
Dayton, Ohio 45433

Chief, Civil Engineering Branch
WERC, Research Division
Air Force Weapons Laboratory
Kirtland AFB, New Mexico 87117

Commander
AFPL (EPNC/Dr. P. H. Kelley)
Edwards AFB, California 93523

Commander
Attn: Mr. A. L. Skinner, COMEAC
Hill AFB, Utah 84401

Commander
Mechanics Division
Air Force Office of Scienc. Res.
Washington, D. C. 20333

NASA

Structures Research Division
Attn: Mr. E. R. Beldenfels, Chief
National Aeronautics & Space Admin.
Langley Research Center
Langley Station
Hampton, Virginia 23365

Chief, Defense Atomic Support Agency.
blast & Shock Division
The Pentagon
Washington, D. C. 20301

Director Defense Research & Engr.
Technical Library
Room 3C-128
The Pentagon
Washington, D. C. 20301

Chief, Airframe & Equipment Branch
PA-120
Office of Flight Standards
Federal Aviation Agency
Washington, D. C. 20553

Chief, Revision of Ship Design
Maritime Administration
Washington, D. C. 20235

Deputy Chief, Office of Ship Constr.
Attn: Mr. G. L. Busey
Maritime Administration
Washington, D. C. 20235

Executive Secretary
Committee on Undersea Warfare
National Academy of Sciences
2101 Constitution Avenue
Washington, D. C. 20418

Ship Hull Research Committee
Attn: Mr. A. R. Lytle
National Research Council
National Academy of Sciences
2101 Constitution Avenue
Washington, D. C. 20418

PART II - CONTRACTORS AND OTHER
TECHNICAL COLLABORATORS

Universities

Dr. D. C. Drucker
Division of Engineering
Brown University
Providence, Rhode Island 02912

Navy (cont'd.)

Undersea Explosion Research Div.
Attn: Mr. D. S. Cohen
Code 780
David Taylor Model Basin
Norfolk Naval Shipyard
Portsmouth, Virginia 23709

Commanding Officer & Director
Code 257, Library
U. S. Navy Marine Engr. Lab.
Annapolis, Maryland 21402

Commander
Technical Library
U. S. Naval Ordnance Test Station
Pasadena Annex
3202 E. Foothill Blvd.
Pasadena, California 91107

U. S. Naval Ordnance Test Station
Attn: Dr. Arnold Adicoff
Code 5056
China Lake, California 93557

Commander
U. S. Naval Ordnance Test Station
Mechanical Engineering Division
Code 556
China Lake, California 93557

Commanding Officer & Director
U. S. Naval Civil Engr. Lab.
Code L31
Port Hueneme, California 93040

Shipyard Technical Library
Code 242L
Portsmouth Naval Shipyard
Portsmouth, New Hampshire 03804

U. S. Naval Electronics Laboratory
Attn: Dr. R. J. Christensen
San Diego, California 92152

U. S. Naval Ordnance Laboratory
Mechanics Division
RFD 1, White Oak
Silver Spring, Maryland 20910

U. S. Naval Ordnance Laboratory
Attn: Mr. H. A. Perry, Jr.
Non-Metallic Materials Division
Silver Spring, Maryland 20910

Supervisor of Shipbuilding
U. S. Navy
Newport News, Virginia 23607

Shipyard Technical Library
Building 746, Code 303YL
Marine Island Naval Shipyard
Vallejo, California 94592

Director
U.S. Navy Underwater Sound Ref. Lab.
Office of Naval Research
P. O. Box 5337
Orlando, Florida 32806

Technical Library
U. S. Naval Propellant Plant
Indian Head, Maryland 20640

U. S. Naval Propellant Plant
Attn: Dr. J. O. Tuono
Research & Development Division
Indian Head, Maryland 20640

Chief of Naval Operations
Attn: Code Op-0380
Op-07T
Department of the Navy
Washington, D. C. 20350

Director, Special Projects
Attn: Sp-001
43
2731
Department of the Navy
Washington, D. C. 20360

Executive Secretary PLRD
Special Projects Office (Sp-00110)
Department of the Navy
Washington, D. C. 20360

U. S. Naval Applied Science Lab.
Code 9832
Technical Library
Building 291, Naval Base
Brooklyn, New York 11251

Director
Aeronautical Materials Lab.
Naval Air Engineering Center
Naval Base
Philadelphia, Pennsylvania 19112

NASA (cont'd.)

National Aeronautics & Space Admin.
Code KV-2
Washington, D. C. 20546

National Aeronautics & Space Admin.
Associate Administrator for Advanced
Research & Technology
Washington, D. C. 20546

Scientific & Tech. Info. Facility
NASA Representative (S-TK/NL)
P. O. Box 5700
Bethesda, Maryland 20801

Other Government Activities

Commandant
Chief, Testing & Development Div.
U. S. Coast Guard
1300 E Street, N. W.
Washington, D. C. 20226

Director
Marine Corps Landing Force Devel. Com.
Marine Corps Schools
Quantico, Virginia 22134

Director
Attn: Mr. B. L. Wilson
National Bureau of Standards
Washington, D. C. 20234

National Science Foundation
Engineering Division
1951 Constitution Avenue, N. W.
Washington, D. C. 20550

Science & Tech. Division
Library of Congress
Washington, D. C. 20540

Director
STNS
Defense Atomic Support Agency
Washington, D. C. 20301

Commander Field Command
Defense Atomic Support Agency
Sandia Base
Albuquerque, New Mexico 87115

Universities (cont'd.)

Prof. M. E. Gurtin
Brown University
Providence, Rhode Island 02912

Prof. R. S. Rivlin
Division of Applied Mathematics
Brown University
Providence, Rhode Island 02912

Prof. P. J. Flata
Materials Science Department
California Institute of Technology
Pasadena, California 91109

Prof. Julius Miklowitz
Div. of Engr. & Applied Sciences
California Institute of Technology
Pasadena, California 91109

Prof. George Sib
Department of Mechanics
Lehigh University
Bethlehem, Pennsylvania 18015

Solid Propellant Library
Firestone Flight Sciences Lab.
California Institute of Technology
Pasadena, California 91109

Prof. Eli Sternberg
Div. of Engr. & Applied Sciences
California Institute of Technology
Pasadena, California 91109

Prof. Paul M. Maghd
Div. of Applied Mechanics
Etcheverry Hall
University of California
Berkeley, California 94720

Prof. J. Baltrukonis
Mechanics Division
The Catholic Univ. of America
Washington, D. C. 20017

Prof. A. J. Duralli
Mechanics Division
The Catholic Univ. of America
Washington, D. C. 20017

Prof. H. H. Kiech
Department of Civil Engr.
Columbia University
Amsterdam & 120th Street
New York, New York 10027

Prof. R. D. Mindlin
Department of Civil Engr.
Columbia University
S. W. Mudd Building
New York, New York 10027

Prof. B. A. Boley
Department of Civil Engr.
Columbia University
Amsterdam & 120th Street
New York, New York 10027

Prof. F. L. DiMaggio
Department of Civil Engr.
Columbia University
616 Mudd Building
New York, New York 10027

Prof. A. M. Freudenthal
Dept. of Civil Engr. & Engr. M
Columbia University
New York, New York 10027

Prof. William A. Nash
Dept. of Engr. Mechanics
University of Florida
Gainesville, Florida 32603

Prof. B. Budiansky
Div. of Engr. & Applied Physics
Pierce Hall
Harvard University
Cambridge, Massachusetts 02138

Prof. P. G. Hodge
Department of Mechanics
Illinois Institute of Technology
Chicago, Illinois 60616

Universities (cont'd.)

Dr. S. L. Koh
School of Aero., Astro. & Engr. Sc.
Purdue University
Lafayette, Indiana 47907

Prof. D. Schapery
Purdue University
Lafayette, Indiana 47907

Prof. E. H. Lee
Div. of Engr. Mechanics
Stanford University
Stanford, California 94305

Dr. Nicholas J. Hoff
Dept. of Aero. & Astro.
Stanford University
Stanford, California 94305

Prof. J. N. Goodier
Div. of Engr. Mechanics
Stanford University
Stanford, California 94305

Prof. Markus Reiner
Technion R & D Foundation, Ltd.
Haifa, Israel

Prof. Tsuyoshi Hayashi
Department of Aeronautics
Faculty of Engineering
University of Tokyo
BUNKYO-KU
Tokyo, Japan

Prof. R. J. H. Bolland
Chairman, Aeronautical Engr. Dept.
207 Guggenheim Hall
University of Washington
Seattle, Washington 98105

Prof. Albert S. Kobayashi
Dept. of Mechanical Engr.
University of Washington
Seattle, Washington 98105

Officer-in-Charge
Post Graduate School for Naval Off.
Webb Institute of Naval Arch.
Crescent Beach Road, Glen Cove
Long Island, New York 11542

Industry and Research Institutes

Mr. K. W. Bills, Jr.
Dept. 4722, Bldg. 0525
Aerojet-General Corporation
P. O. Box 1947
Sacramento, California 95809

Dr. James H. Wiegand
Senior Dept. 4720, Bldg. 0525
Ballistics & Mech. Properties Lab.
Aerojet-General Corporation
P. O. Box 1947
Sacramento, California 95809

Dr. John Zickel
Dept. 4650, Bldg. 0227
Aerojet-General Corporation
P. O. Box 1947
Sacramento, California 95809

Mr. J. S. Wise
Aerospace Corporation
P. O. Box 1308
San Bernardino, California 92402

Dr. Vito Salerno
Applied Technology Assoc., Inc.
29 Church Street
Ramsey, New Jersey 07446

Library Services Department
Report Section, Bldg. 18-14
Argonne National Laboratory
9700 S. Cass Avenue
Argonne, Illinois 60440

Dr. E. M. Kerwin
Bolt, Beranek, & Newman, Inc.
50 Moulton Street
Cambridge, Massachusetts 02138

Dr. M. C. Junger
Cambridge Acoustical Associates
129 Mount Auburn Street
Cambridge, Massachusetts 02138

Dr. F. R. Schwarzl
Central Laboratory T.N.C.
134 Julianalaan
Delft, Holland

Universities (cont'd.)

Prof. H. T. Corten
University of Illinois
Urbana, Illinois 61803

Prof. W. J. Hall
Department of Civil Engr.
University of Illinois
Urbana, Illinois 61803

Prof. W. M. Newark
Dept. of Civil Engineering
University of Illinois
Urbana, Illinois 61803

Dr. W. H. Avery
Applied Physics Laboratory
Johns Hopkins University
3621 Georgia Avenue
Silver Spring, Maryland 20910

Prof. J. B. Tiedemann
Dept. of Aero. Engr. & Arch.
University of Kansas
Lawrence, Kansas 66045

Prof. S. Taira
Department of Engineering
Kyoto University
Kyoto, Japan

Prof. E. Weissner
Dept. of Mathematics
Massachusetts Inst. of Tech.
Cambridge, Massachusetts 02139

Library (Code 0384)
U. S. Naval Postgraduate School
Monterey, California 93940

Dr. Joseph Marin
Prof. of Materials Science
Dept. of Materials Sc. & Chem.
U. S. Naval Postgraduate School
Monterey, California 93940

Prof. E. L. Reiss
Courant Inst. of Math. Sciences
New York University
4 Washington Place
New York, New York 10003

Dr. Francis Cozzarelli
Div. of Interdisciplinary
Studies and Research
School of Engineering
State Univ. of N.Y. at Buffalo
Buffalo, New York 14214

Dr. George Herrmann
The Technological Institute
Northwestern University
Evanston, Illinois 60201

Director, Ordnance Research Lab.
The Pennsylvania State University
P. O. Box 30
State College, Pennsylvania 16801

Prof. Eugen J. Skudrzyk
Department of Physics
Ordnance Research Lab.
The Pennsylvania State University
P. O. Box 30
State College, Pennsylvania 16801

Dean Oscar Baguio
Assoc. of Structural Engr.
of the Philippines
University of Philippines
Manila, Philippines

Prof. J. Kempner
Dept. of Aero. Engr. & Applied Mech.
Polytechnic Institute of Brooklyn
333 Jay Street
Brooklyn, New York 11201

Prof. J. Klosner
Rytechnic Institute of Brooklyn
333 Jay Street
Brooklyn, New York 11201

Prof. F. R. Eirich
Polytechnic Institute of Brooklyn
333 Jay Street
Brooklyn, New York 11201

Prof. A. C. Eringen
School of Aero., Astro. & Engr. Sc.
Purdue University
Lafayette, Indiana 47907

Industry & Research Inst. (cont'd.)

Mr. Ronald D. Brown
Infrared Physics Laboratory
Chemical Precision Agency
5621 Georgia Avenue
Silver Spring, Maryland 20910

Research and Development
Electric Boat Division
General Dynamics Corporation
Groton, Connecticut 06340

Supervisor of Shipbuilding, USN,
and Naval Insp. of Ordnance
Electric Boat Division
General Dynamics Corporation
Groton, Connecticut 06340

Dr. L. H. Chen
Basic Engineering
Electric Boat Division
General Dynamics Corporation
Groton, Connecticut 06340

Mr. Ross H. Petty
Technical Librarian
Allegany Ballistic Laboratory
Hercules Powder Company
P. O. Box 210
Cumberland, Maryland 21501

Dr. J. H. Thacher
Allegany Ballistic Laboratory
Hercules Powder Company
Cumberland, Maryland 21501

Dr. Joshua E. Greenspan
J. E. Engr. Research Associates
3831 Meado Drive
Baltimore, Maryland 21215

Mr. R. F. Landel
Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, California 91103

Mr. G. Lewis
Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, California 91103

Dr. M. L. Merritt
Division 5112
Sandia Corporation
Sandia Base
Albuquerque, New Mexico 87115

Director
Ship Research Institute
Ministry of Transportation
700, SHINKAWA
Mitaka
Tokyo, JAPAN

Dr. H. N. Abramson
Southeast Research Institute
8500 Culvers Road
San Antonio, Texas 78206

Industry & Research Inst. (cont'd.)

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78206

Dr. Thor Smith
Stanford Research Institute
Menlo Park, California 94025

Dr. M. L. Baron
Paul Weidlinger, Consulting Engr.
777 Third Ave. - 22nd Floor
New York, New York 10017

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Department of Mechanics Lehigh University		2a. REPORT SECURITY CLASSIFICATION Unclassified
2b. GROUP		
3. REPORT TITLE On the Westergaard Method of Crack Analysis		
4. DESCRIPTIVE NOTES (Type of report and Inclusive dates) Research Project		
5. AUTHOR(S) (Last name, First name, Middle) Sih, G. C.		
6. REPORT DATE March 1966	7a. TOTAL NO. OF PAGES 7	7b. NO. OF REPS 3
8a. CONTRACT OR GRANT NO. Nonr-610(06)	8c. ORIGINATOR'S REPORT NUMBER(S) No. 1	
a. PROJECT NO. NR-064-476	9a. OTHER REPORT NO(S) (Any other numbers that may be assigned to this report)	
b.	d.	
10. AVAILABILITY/LIMITATION NOTICES Reproduction in whole or in part is permitted by the United States Government. Distribution of this document is unlimited.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Office of Naval Research	
13. ABSTRACT <p>The Westergaard method of crack analysis, published almost thirty years ago, is shown to be invalid for a class of crack problems dealing with the infinite medium with cracks under applied loads at infinity. The necessary modifications of the Westergaard method are derived from the complex potential formulation of Muskhelishvili. The examples of a single line crack in an infinite plate owing to biaxial tension and pure shear are discussed.</p>		

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fracture mechanics Cracks Plane extension						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.
- 2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.
- 2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.
3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.
4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.
5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.
6. REPORT DATE: Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.
- 7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.
- 7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.
- 8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.
- 8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system number, task number, etc.
- 9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.
- 9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).
10. AVAILABILITY/LIMITATION NOTICE: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

- (1) "Qualified requesters may obtain copies of this report from DDC."
- (2) "Foreign announcement and dissemination of this report by DDC is not authorized."
- (3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ."
- (4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ."
- (5) "All distribution of this report is controlled. Qualified DDC users shall request through ."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.